CALCULATION SHEET

By: JOH

Checked:____

PROJECT: Threesome

DATE: 25/01/07

SUBJECT: Brake rim heating

BY: Jake Hannah

HEAT GAINED BY THE FRONT BRAKING RIM OF A TRANDEM UNDER BRAKING

1.0 INTRODUCTION

1.1 Purpose

The purpose of this calculation is to find the temperature of the front rim of a trandem bicycle under braking.

1.2 Scope

Heat gain is calculated when descending on a steady slope, with a given initial velocity to a halt. Energy losses due to wind resistance and rolling resistance are considered.

1.3 Assumptions

- Deceleration is considered to be constant, it so much easier.
- The braking effort is split 70:30 between the front and back wheels. This is the accepted norm for a standard bicycle. Christ knows for a trandem.
- The energy created at the brake pad/rim interface is absorbed half and half between the pad and the rim. No-one seems to know about this one and loads of other people make this rather random assumption.
- No cooling of the rims is allowed for as it is feckin hard to model. There would be some, particularly travelling at speed. In attempt to minimise the cooling effect, relatively short times are considered.
- Deceleration is calculated to bring the bike (and riders hopefully) to rest.

2.0 INPUT DATA

2.1 Constants

Imperial inch	$inch \equiv 25.4mm$
Degree Centrigrade (scalar)	$degC \equiv 1K$
Acceleration due to gravity	$g = 9.807\,m \cdot s^{-2}$

2.2 Bike and Riders

Mass of each rider	$m_{rider} := 85kg$
Mass of bike	$m_{bike} := 30kg$
Frontal surface area	$A := 0.5m^2$
Initial speed of bike	$u := 50km \cdot hr^{-1}$
Final speed of bike	$v := 0km \cdot hr^{-1}$
Time to stop	$t := 60s$
Slope angle (+ve downhill)	$\theta := 5deg$

2.3 Rim

Diameter of braking rim	$D := 26inch$
Height of rim	$h_{rim} := 10mm$
Thickness of rim	$t_{rim} := 1mm$
Density of steel	$\rho_{steel} := 7850kg \cdot m^{-3}$
Specific heat capacity of steel	$c_{steel} := 450J \cdot kg^{-1} \cdot K^{-1}$
Initial temperature of rim	$T_i := 25degC$

2.4 Rolling resistance

Coefficient of rolling resistance	$c_r := 0.004$

2.5 Wind resistance

Coefficient of wind resistance	$c_w := 0.5$
Density of air	$\rho_{air} := 1.226kg \cdot m^{-3}$

3.0 CALCULATIONS

3.1 Stopping distance

Required constant deceleration

$a := \frac{v - u}{t}$ $\quad a = -0.231 \frac{m}{s^2}$

Distance taken to stop

$d := \frac{1}{2}(u + v)\,t$ $\quad d = 416.667\,m$

3.2 Energy of bike and rider

Total mass of bike and riders

$m_{total} := m_{bike} + 3m_{rider}$ $\quad m_{total} = 285\,kg$

Initial kinetic energy of the crew

$ke := \frac{1}{2} \cdot m_{total} \cdot u^2$ $\quad ke = 2.749 \times 10^4\,J$

Gravitational potential energy gained

$e_g := m_{total} \cdot g \cdot d \cdot \sin(\theta)$ $\quad e_g = 1.015 \times 10^5\,J$

3.3 Rolling resistance

Retardinf force due to rolling resistance

$F_r := c_r \cdot m_{total} \cdot g$ $\quad F_r = 11.18\,N$

Total loss of energy due to rolling resistance

$e_r := F_r \cdot d$ $\quad e_r = 4.658 \times 10^3\,J$

3.4 Wind resistance

Force due to wind resistance

$F_w := \frac{1}{2} c_w \cdot A \cdot \rho_{air} \cdot u^2$

Velocity, u expressed in terms of distance (equations of motion)

$u := \sqrt{v^2 - 2a \cdot d}$ where v=0

Therefore the wind resistance can be expressed as

$$F_w := \frac{1}{2} c_w \cdot A \cdot \rho_{air} \cdot (-2a \cdot d)$$

Integrating the equation for wind force with respect to distance covered, assuming deceleration is constant, gives the total energy lost during due to wind loss.

$$e_w := \int_0^d \left[\frac{1}{2} \cdot c_w \cdot A \cdot \rho_{air} \cdot (-2 \cdot a \cdot x)\right] dx \qquad e_w = 6.159 \times 10^3 \, J$$

3.5 Mass of rims

Circumference of rim

$$p_{rim} := \pi D \qquad p_{rim} = 2.075 \, m$$

Volume of steel in rim

$$V_{rim} := p_{rim} \cdot h_{rim} \cdot t_{rim} \qquad V_{rim} = 2.075 \times 10^{-5} \, m^3$$

Mass of both rims

$$m_{rim} := 2 \cdot \rho_{steel} \cdot V_{rim} \qquad m_{rim} = 0.326 \, kg$$

3.6 Heat gained by rims

Energy absorbed as heat by brakes and rims

$$e_b := ke + e_g - e_r - e_w \qquad e_b = 1.182 \times 10^5 \, J$$

Energy absorbed by front rims, assuming a 50:50 split with the pad and a 70:30 front/back wheel split

$$e_{rim} := 0.7 \frac{e_b}{2} \qquad e_{rim} = 4.136 \times 10^4 \, J$$

Change in temperature of the rims

$$\Delta T := \frac{e_{rim}}{m_{rim} \cdot c_{steel}} \qquad \Delta T = 282.162 \, degC$$

Final temperature of the rims

$$T_f := \Delta T + T_i \qquad T_f = 307.162 \, degC$$